Componental specification of plane waves in isotropic and anisotropic viscoelastic media

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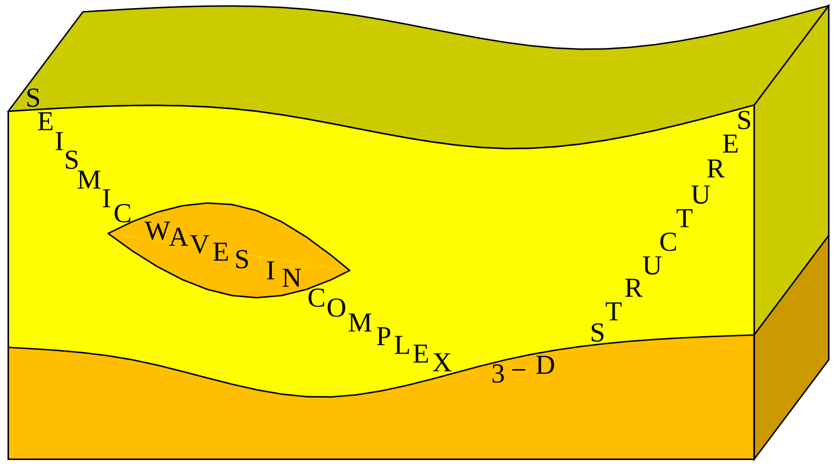

Displacement vector of a harmonic plane wave

$$\mathbf{u}(\mathbf{x}, t) = a\mathbf{U}\exp\left(-\mathrm{i}\omega(t - \mathbf{p}\cdot\mathbf{x})\right)$$

$\mathbf{u}$ - displacement vector

$\mathbf{x}$ - position vector

t - time

ω - circular frequency

a - scalar amplitude factor

$\mathbf{U}$ - normalized polarization vector ($\mathbf{U}\cdot\mathbf{U} = 1$)

$\mathbf{p}$ - slowness vector

Slowness vector (generally complex valued)

$\mathbf{p} = \mathbf{P} + \mathrm{i}\mathbf{A}$

$\mathbf{P}$ - propagation vector, $\mathbf{A}$ - attenuation vector

Attenuation angle γ

$\cos\gamma = \mathbf{P} \cdot \mathbf{A} / |\mathbf{P}||\mathbf{A}|$

$\gamma = \mathbf{0}$ - homogeneous wave

$\gamma \neq \mathbf{0}$ - inhomogeneous wave

Some choices of $\mathbf{P}$ and $\mathbf{A}$ nonphysical (forbidden directions)

Two specifications of the slowness vector

Componental and mixed specifications

- avoid problems with "forbidden directions"
- obtained by solving polynomial equation of 6th degree

Componental specification

- useful for study of R/T problems
- problems with selection of signs of complex square roots

Mixed specification

- useful in unbounded media
- no problems with selection of signs of complex square roots

Componental specification of slowness vector

$$\mathbf{p} = \sigma^{\Sigma}\mathbf{n}^{\Sigma} + \mathbf{p}^{\Sigma}, \qquad \mathbf{n}^{\Sigma} \cdot \mathbf{p}^{\Sigma} = 0$$

$\mathbf{n}^{\Sigma}$ - unit normal to Σ (given; real valued)

$\mathbf{p}^{\Sigma}$ - projection of $\mathbf{p}$ to Σ (given; complex valued)

each combination of $\mathbf{n}^{\Sigma}$ and $\mathbf{p}^{\Sigma}$ specifies uniquely an existing plane wave

σ^{Σ} - complex-valued scalar to be determined

Determination of the complex-valued scalar

$$\det[a_{ijkl}(\sigma^{\Sigma} n_j^{\Sigma} + p_j^{\Sigma})(\sigma^{\Sigma} n_l^{\Sigma} + p_l^{\Sigma}) - \delta_{ik}] = 0 \ (*)$$

(*) - polynomial equation of the 6th degree

for complex-valued σ^{Σ}

a_{ijkl} - density-normalized elastic or viscoelastic moduli

Mixed specification of slowness vector

$$\mathbf{p} = \sigma\mathbf{n} + \mathrm{i}D\mathbf{m}, \qquad \mathbf{n}\cdot\mathbf{m} = 0, \qquad \mathbf{n} \parallel \mathbf{P} \qquad (\mathbf{p}^{\Sigma} = \mathrm{i}D\mathbf{m})$$

$\mathbf{n}$, $\mathbf{m}$ - unit vectors (given; real valued)

D - inhomogeneity parameter (given; real valued)

each combination of $\mathbf{n}$, $\mathbf{m}$ and D specifies uniquely an existing plane wave

σ - complex-valued scalar to be determined

$D = 0$ - homogeneous wave $\qquad$ $D \neq 0$ - inhomogeneous wave

Determination of the complex-valued scalar

$$\det[a_{ijkl}(\sigma n_j + \mathrm{i}Dm_j)(\sigma n_l + \mathrm{i}Dm_l) - \delta_{ik}] = 0 \ (*)$$

(*) - polynomial equation of the 6th degree
for complex-valued σ

a_{ijkl} - density-normalized elastic or viscoelastic moduli

Componental versus mixed specifications

Componental → mixed: $\mathbf{p} = \sigma^{\Sigma}\mathbf{n}^{\Sigma} + \mathbf{p}^{\Sigma} \rightarrow \mathbf{p} = \sigma\mathbf{n} + \mathrm{i}D\mathbf{m}$

$$\mathbf{n} = \mathbf{P}/|\mathbf{P}|, \qquad \sigma = (\sigma^{\Sigma}\mathbf{n}^{\Sigma} + \mathbf{p}^{\Sigma}) \cdot \mathbf{n}$$

$$\mathbf{m} = \mathrm{Im}(\sigma^{\Sigma}\mathbf{n}^{\Sigma} - \sigma\mathbf{n} + \mathbf{p}^{\Sigma})/|\mathrm{Im}(\sigma^{\Sigma}\mathbf{n}^{\Sigma} - \sigma\mathbf{n} + \mathbf{p}^{\Sigma})|$$

$$D = |\mathrm{Im}(\sigma^{\Sigma}\mathbf{n}^{\Sigma} - \sigma\mathbf{n} + \mathbf{p}^{\Sigma})|$$

Componental versus mixed specifications

Mixed $\rightarrow$ componental: $\mathbf{p} = \sigma\mathbf{n} + \mathrm{i}D\mathbf{m} \longrightarrow \mathbf{p} = \sigma^{\Sigma}\mathbf{n}^{\Sigma} + \mathbf{p}^{\Sigma}$

$\mathbf{n}^{\Sigma}$ - unit normal to Σ

$\sigma^{\Sigma} = (\sigma\mathbf{n} + \mathrm{i}D\mathbf{m}) \cdot \mathbf{n}^{\Sigma}$

$\mathbf{p}^{\Sigma} = \sigma\mathbf{n}^{\Sigma} - \sigma^{\Sigma}\mathbf{n}^{\Sigma} + \mathrm{i}D\mathbf{m}$

R/T in anisotropic viscoelastic media

Incident wave: $\mathbf{p} = \sigma\mathbf{n} + \mathrm{i}D\mathbf{m}$

$$\det[a^{inc}_{ijkl}(\sigma n_j + \mathrm{i}Dm_j)(\sigma n_l + \mathrm{i}Dm_l) - \delta_{ik}] = 0$$

Generated wave: $\mathbf{p}^g = \sigma^g\mathbf{n}^\Sigma + \mathbf{p}^\Sigma$

$$\mathbf{p}^\Sigma = \sigma\mathbf{N}^\Sigma + \mathrm{i}D\mathbf{M}^\Sigma$$

$$\mathbf{N}^\Sigma = \mathbf{n}^\Sigma \times (\mathbf{n} \times \mathbf{n}^\Sigma), \qquad \mathbf{M}^\Sigma = \mathbf{n}^\Sigma \times (\mathbf{m} \times \mathbf{n}^\Sigma)$$

$$\det[a^{g}_{ijkl}(\sigma^g n^\Sigma_j + p^\Sigma_j)(\sigma^g n^\Sigma_l + p^\Sigma_l) - \delta_{ik}] = 0$$

Studied special cases

- special choices of $\mathbf{p}^{\Sigma}$:

  real valued, imaginary valued, $\mathrm{Im}\mathbf{p}^{\Sigma} \parallel \mathrm{Re}\mathbf{p}^{\Sigma}$, etc

- special cases of media:

  isotropic viscoelastic, monoclinic viscoelastic, etc

- types of waves:

  P and S, SH, inhomogeneous, homogeneous, etc